quadratically_constrained_quadratic_bilevel

Let $x \in \mathbb{R}^{n_x}, y \in \mathbb{R}^{n_y}$ be the lower and upper level decision variables, respectively. Let $c \in \mathbb{R}^{n_x}$, $d \in \mathbb{R}^{n_y}$ be data vectors and $P \in \mathbb{R}^{n_x \times n_x}$, $Q \in \mathbb{R}^{n_y \times n_y}$, $R \in \mathbb{R}^{n_x \times n_y}$ data matrices and $s \in \mathbb{R}$ a scalar then the *Quadratic Expression* in x, y is defined as:

$$\mathcal{Q}_{c,d,P,Q,R,s}(x, y) \coloneqq \frac{1}{2} \begin{bmatrix} x \\ y \end{bmatrix}^\top \begin{bmatrix} P & R \\ R^\top & Q \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}^\top \begin{bmatrix} x \\ y \end{bmatrix} + s.$$

Given a sequence of quadratic expression $\mathcal{Q}^0_{\text{up}}, \mathcal{Q}^k_{\text{up}}, \mathcal{Q}^0_{\text{lo}}, \mathcal{Q}^k_{\text{lo}}$ the *Quadratically Constrained Quadratic Bilevel Program* (QCQBP) over x, y is defined as:

$$\begin{aligned} \underset{x \in \mathbb{R}^{n_x}, y \in \mathbb{R}^{n_y}}{\text{minimise}} \quad & \mathcal{Q}^0_{\text{up}}(x, y) \\ \text{subject to} \quad & \mathcal{Q}^k_{\text{up}}(x, y) \geq 0, \qquad \text{for } k = 1, 2, \ldots, \\ & A_{\text{up}} x + B_{\text{up}} y \geq r_{\text{up}}, \\ & \underline{x} \leq x \leq \bar{x}, \\ & y \in \underset{y \in \mathbb{R}^{n_y}}{\arg\min} \begin{cases} \mathcal{Q}^0_{\text{lo}}(x, y) \\ \text{s.t.} \quad \mathcal{Q}^k_{\text{lo}}(x, y) \geq 0, \qquad \text{for } k = 1, 2, \ldots, \\ \qquad A_{\text{lo}} x + B_{\text{lo}} y \geq r_{\text{lo}}, \\ \qquad \underline{y} \leq y \leq \bar{y}. \end{cases} \end{aligned} \tag{QCQBP}$$

We store the parameters of each quadratic expression (c, d, P, Q, R, s) as named vectors and matrices in a JSON file. For example consider the (QCQBP) instance given by Allende and Still [1, page 331]:

$$\begin{aligned} \underset{x \in \mathbb{R}^{n_x}, y \in \mathbb{R}^{n_y}}{\text{minimise}} \quad & F(x, y) \coloneqq -2x_1 - 2x_2 - x_1^2 + x_2^2 + y_1^2 + y_2^2 \\ \text{subject to} \quad & x \geq 0, \quad y \geq 0, \quad -x_1 + 2 \geq 0, \\ & y \in \underset{y \in \mathbb{R}^{n_y}}{\arg\min} \begin{cases} y_1^2 - 2x_1 y_1 + y_2^2 - 2x_2 y_2 \\ \text{s.t.} \quad 0.25 - (y_1 - 1)^2 \geq 0, \\ \qquad 0.25 - (y_2 - 1)^2 \geq 0. \end{cases} \end{aligned}$$

The upper-level objective is a quadratic expression whose coefficients are stored in the following way:

```
"F": {
    "c": [-2, -2],
    "d": [0, 0],
    "P": [[-2, 0], [0, 2]],
    "Q": [[2, 0], [0, 2]],
    "R": [[0, 0], [0, 0]],
    "s": 0
},
```

References

[1] Gemayqzel Bouza Allende and Georg Still. Solving bilevel programs with the kkt-approach. *Mathematical programming*, 138(1):309–332, 2013.