dantzig_3_3

This transportation problem was first introduced by Dantzig [1, Chapter 3.3, page 35] The cities of Seattle and San Diego have a supply of 350 and 650 units respectively. The cities New York, Chicago and Topeka have a variable demand of x_1, x_2 and x_3 units respectively. The lower-level variables $y \in \mathbb{R}^6$ represent the transportation between each pair of cities, which each incurs a fixed cost represented in the table below:

	New York	Chicago	Topeka
Seattle	y_1 (\$2.5)	y_2 (\$1.7)	y_3 (\$1.8)
San Diego	y_4 (\$2.5)	y_5 (\$1.8)	y_6 (\$1.4)

The lower-level program aims to find the minimal transportation cost while meeting supply-demand constraints.

$$\begin{array}{ll} \underset{y}{\text{minimise}} & 2.5y_1 + 1.7y_2 + 1.8y_3 + 2.5y_4 + 1.8y_5 + 1.4y_6 \\ \text{subject to} & \left\{ \begin{array}{ccccccll} y_1 & +y_2 & +y_3 & & & & \leq & 350 \\ & & & y_4 & +y_5 & +y_6 & \leq & 650 \\ y_1 & & & y_4 & & & \geq & x_1 \\ & y_2 & & & y_5 & & \geq & x_2 \\ & & y_3 & & & y_6 & \geq & x_3 \end{array} \right. \end{array}$$

The upper-level program tries to arrange a demand as close to 400 units as possible for New York, Chicago and Topeka while still maintaining a minimal transportation cost.

$$\begin{array}{ll} \underset{x_1,x_2,x_3,y}{\text{minimise}} & (x_1 - 400)^2 + (x_2 - 400)^2 + (x_2 - 400)^2 \\ \text{subject to} & x_1 \geq 325, \quad x_2 \geq 300, \quad x_3 \geq 275, \\ & y \text{ solves lower-level} \end{array}$$

			Dimension	Type
Upper-level	x	variables	3	real
	F(x,y)	objective	1	sum of squares
	G(x,y)	inequality	3	bounds
	H(x,y)	equality	0	none
Lower-level	y	variables	6	real
	f(x,y)	objective	1	linear
	g(x,y)	inequality	5	linear
	h(x,y)	equality	0	none

References

[1] George B. Dantzig. *Linear Programming and Extensions.* Princeton University Press, 1963.