brotcorne2001

Let $G = (\mathcal{N}, \mathcal{A})$ be a transportation network where $\mathcal{N}$ denotes the set of nodes and $\mathcal{A}$ the set of arcs. With each arc a is associated a fixed travel cost c_a and an additional variable toll T_a. Let $\mathcal{K}$ denote the set of commodities. Each node $i \in \mathcal{N}$ has a supply-demand d_i^k for commodity k.

Variable	Description	Type
T_a	Toll on arc a	Upper-level variable
y_a^k	Flow of commodity k along arc a	Lower-level variable
b_i^k	Supply of/demand for commodity k at node i	Data
c_a	Cost per flow of arc a	Data
$T_a^{\max}$	The maximum toll on arc a	Data

The leader's objective is to maximize the total revenue, which is the sum of the products between toll T_a and the number of users on arc a. The objective of the lower-level problem is to meet the supply-demand constraint while minimising the total cost of paths selected by the network users. [1]

$$
\begin{aligned}
\underset{T,y}{\text{maximise}} \quad & \sum_{a \in \mathcal{A}} T_a \sum_{a \in \mathcal{K}} y_a^k \\
\text{subject to} \quad & T_a \leq T_a^{\max} \quad \forall a \in \mathcal{A} \\
& y \in \underset{y}{\arg\min} \quad \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}} (c_a + T_a) y_a^k \\
& \qquad \text{subject to} \quad \sum_{a \in i^+} y_a^k - \sum_{a \in i^-} y_a^k = b_i^k \quad \forall k \in \mathcal{K},\ \forall i \in \mathcal{N}, \\
& \qquad\qquad\qquad\quad y_a^k \geq 0 \quad \forall k \in \mathcal{K},\ \forall a \in \mathcal{A},
\end{aligned}
$$

			Dimension	Type
Upper-level	x	variables	7	real
	F(x,y)	objective	1	non-convex
	G(x,y)	inequality	14	bounds
	H(x,y)	equality	0	none
Lower-level	y	variables	7	real
	f(x,y)	objective	1	linear
	g(x,y)	inequality	7	bounds
	h(x,y)	equality	6	linear

References

[1] Luce Brotcorne, Martine Labbé, Patrice Marcotte, and Gilles Savard. A bilevel model for toll optimization on a multicommodity transportation network. *Transportation Science*, 35(4):345–358, 2001.