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With a quartic lower-level objective function, this example remains convex but cannot be solved with quadratic methods. [1, Chapter 8, example 8.7.1, page 358]. It has a global solution at $(x, y) = (11.25, 5)$ and a local solution at $(x, y) = (7.2, 12.8)$.

$$
\begin{aligned}
\underset{x}{\text{minimise}} \quad & 16x^2 + 9y^{*2} \\
\text{subject to} \quad & x \geq 0, \\
& 4x - y^* \geq 0, \\
& y^* \in \underset{y}{\arg\min} \left\{ \begin{array}{ll} (x+y-20)^4 & \\ \text{subject to} & -4x - y + 50 \geq 0, \\ & y \geq 0. \end{array} \right.
\end{aligned}
$$

			Dimension	Type
Upper-level	x	variables	1	real
	F(x,y)	objective	1	quadratic
	G(x,y)	inequality	2	linear
	H(x,y)	equality	0	none
Lower-level	y	variables	1	real
	f(x,y)	objective	1	convex
	g(x,y)	inequality	2	linear
	h(x,y)	equality	0	none

References

[1] Jonathan Bard. *Practical Bilevel Optimization: Algorithms And Applications*, volume 30. Springer Science & Business Media, 09 1998.