bard851

This example was first presented in [2, Fig. 1] and then later in [1, Chapter 8, example 8.5.1, page 336]. The feasible set is a polyhedron with a global optimum at $(x, y_1, y_2) = (\frac{17}{9}, \frac{8}{9}, 0)$.

$$\begin{aligned} \underset{x, y_1^*, y_2^*}{\text{minimise}} \quad & (x-1)^2 + 2y_1^{*2} - 2x \\ \text{subject to} \quad & x \geq 0 \\ & (y_1^*, y_2^*) \in \underset{y_1, y_2}{\arg\min} \begin{cases} (2y_1 - 4)^2 + (2y_2 - 1)^2 + xy_1 \\ \text{s.t.} \quad 4x + 5y_1 + 4y_2 \leq 12 \\ \quad\quad -4x + 5y_1 + 4y_2 \leq -4 \\ \quad\quad 4x - 4y_1 + 5y_2 \leq 4 \\ \quad\quad -4x + 4y_1 + 5y_2 \leq -4 \\ \quad\quad y_1, y_2 \geq 0 \end{cases} \end{aligned}$$

			Dimension	Type
Upper-level	x	variables	1	real
	F(x,y)	objective	1	quadratic
	G(x,y)	inequality	1	bounds
	H(x,y)	equality	0	none
Lower-level	y	variables	2	real
	f(x,y)	objective	1	quadratic
	g(x,y)	inequality	6	linear
	h(x,y)	equality	0	none

References

[1] Jonathan Bard. *Practical Bilevel Optimization: Algorithms And Applications*, volume 30. Springer Science & Business Media, 09 1998.

[2] Gilles Savard and Jacques Gauvin. The steepest descent direction for the nonlinear bilevel programming problem. *Operations Research Letters*, 15(5):265–272, 1994.